Sated but Thirsty

Towards a Multidimensional Measure of Need-Based Justice

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Abstract: Measures of need-based justice that have been proposed lately rely on a single dimension of need that is taken into account. This is shown to be problematic since humans experience different kinds of need that appear to be incommensurable.

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1 Introduction

Need plays an important role for considerations on distributive justice, being one of few major categories that are generally considered relevant in this area (Forsyth 2006). There are several attempts on measuring the justice of some given allocation; often with a focus on inequality (Gini 1914, Atkinson 1970, Firebaugh 1999, Lambert 2001). From a sociological point of view, Jasso (1999, 2007, Jasso and Wegener 1997) prominently suggests general metrics of (perceived) justice. In an early paper Jasso (1978) refers to a number of other suggestions by Adams (1965), Berger and colleagues (1972), Homans (1974), as well as Walster and colleagues (1976). Eriksson (2012) discusses further rudimentary metrics referring to Jasso. Additionally, Braß (1994) considers problems of fair representation as a mathematical problem of distributive justice.

Attempts of measuring justice with respect to need are relatively scarce, including a rudimentary approach by Miller (1999), and considerations by Hassoun (2009). Bauer (2017a, b) suggests a modified index that is borrowed from the measurement of poverty, Siebel (2017) recalls on concepts from classical antiquity for this purpose, Traub and colleagues (2017) introduce an index that satisfies a set of axioms proposed to be important for the measurement of need-based justice, and Springhorn (2017) introduces a measure that focuses on the perspective of a single individual within a given distribution.

Those attempts in general – though being very divers – propose a one-dimensional reference point; a person gets attributed a single value quantifying its legitimate claim and a single value representing its actual stake. Take for example a one-dimensional measure (here denoted \( O \)) from Jasso (1999) that can be stated as follows:

\[
O_{\text{Jasso}}(x, y) = \frac{1}{n} \sum_{i=1}^{n} \ln \left( \frac{x_i}{y_i} \right)
\]  

(1)

In (1) she suggests the arithmetic mean for the aggregation of the justice evaluation functions from several individuals \( i \). Those functions consist of the natural logarithm of the quotient of some normative target value \( y \) and the actual allocation \( x \) of the individual \( i \). To name another recent example, Bauer (2017) states a measure that is a modified version of a widely used poverty index stated by Foster and colleagues (1984):
\[
O_{\text{Bauer}}(x, y) = \frac{u}{p} \sum_{i \in U} \left( \frac{(y_i - x_i)}{x} \right)^{\alpha} + \frac{o}{p} \sum_{i \in O} \left( \frac{(x_i - y_i)}{y} \right)^{\beta}
\] (2)

In (2) it is suggested to take the oversupplied \((O)\) and undersupplied \((U)\) individuals into account separately using two different sums, weighted by their share of the whole population \((P)\). Some powers \((\alpha, \beta)\) are given as parameters of aversion or affinity against or for under- or oversupply.

Such a reduction to a single dimension – in this case \(y_i\) as the single normative reference point for the need of some individual \(i\) and \(x_i\) as the corresponding allocation – appears to be problematic in the case of needs-based justice.\(^1\) The category of need is – in stark contrast to the models recently proposed – a multidimensional one that can hardly be broken down into a single dimensional value. People have a variety of needs and there appears to be some kind of incommensurability between those.

The need for water and the need for food, to choose a simple example from the class of biologically motivated basic needs, both count (probably without question) as legitimate. Nonetheless, it appears to be problematic to attribute a single value of need to a person that feels both hunger and thirst, because those needs usually demand different goods for their satisfaction. This assumed, summing up two numbers that represent a persons need for hydration and her need for nutrition leads to problems: Take for example a need of, say, five units of drinking to satisfy a persons need for hydration and an additional need of, say, five units of nourishment to satisfy her need for nutrition, adding up to a total need of ten units to satisfy this persons need. Now assume that this individual receives a total of ten units of food. If one solely relies on the aggregated, dimensionless values of needing ten units and receiving ten units, the person would seem to be satisfied – though even unlimited supplies of some kind of food may not be able to meet her demands of hydration.

\(^1\)Economic research on poverty measurement has thus put forth the use of net equivalent incomes or multidimensional approaches (Kapteyn and van Praag 1976, van Praag 1977, Atkinson and Bourguignon 1987, Ebert and Moyes 2003, Jenkins and Lambert 2005). Some approaches, for example, make use of subjective poverty lines (Goedhart et al. 1977, Flink and van Praag 1991), others try explicitly to be multidimensional (Bourguignon and Chakravarty 2003, Alkire and Foster 2011, Kakwani and Silber 2008). There is also measurement of inequality and welfare with heterogeneous needs, where amongst others Atkinson and Bourguignon (1987) have to be mentioned, as well as Lambert and Ramos (2002, Chakravarty 2009).
Considering, for example, Maslow’s (1943) proposed hierarchy of needs, this problem can easily be extended to further categories of need. This should be taken into account when constructing a measure of need-based justice. Therefore, a first suggestion on how to tackle this problem is to be introduced in the following.

2 Notation and Definitions

First of all a formal notation is to be introduced that has to include those aspects that are considered as relevant. This of course already requires a selection that never is free from normative assumptions.

First of all, a set $\mathcal{P}$ of individuals $i = \{1, \ldots, n\}$ is considered, their number is given by $p = \#(\mathcal{P})$. Those individuals do not have to represent singular persons, they also can describe groups, for example households or institutions.

It is assumed that every individual $i$ exhibits need in several dimensions $j = \{1, \ldots, m\}$, giving us $\nu_i^j$ that is to be quantified within the nonnegative real numbers, $\nu_i^j \in \mathbb{R}_0^+$. For some individual $i$ one can then denote $\nu_i^1$ to $\nu_i^m$ needs. For a set $\mathcal{P}$ of individuals $i = \{1, \ldots, n\}$ this gives us a matrix $N$ of needs for all $\nu_i^j$ as $N = (\nu_i^j)$ with $\nu_i^j > 0$ for $j = \{1, \ldots, m\}$ and $i = \{1, \ldots, n\}$.

\[
N = \begin{pmatrix} \nu_1^1 & \nu_2^1 & \cdots & \nu_n^1 \\ \nu_1^2 & \nu_2^2 & \cdots & \nu_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \nu_1^m & \nu_2^m & \cdots & \nu_n^m \end{pmatrix}
\]

Furthermore it is assumed that every individual $i$ is independently from its needs endowed with actual allocations of some goods $\gamma_i^j$ for each need, also being quantified within the nonnegative real numbers, $\gamma_i^j \in \mathbb{R}_0^+$. The allocated goods don’t have to be limited to physical goods, they nonetheless have to be quantifiable.

It can be assumed that there are different goods for the satisfaction of different needs, for simplification also consisting of dimensions $j = \{1, \ldots, m\}$ representing $m$ dimensions of goods for the $m$ dimensions of needs. This way it is not taken into account that sometimes several goods may satisfy the same needs or that the same good can satisfy different needs. A matrix $G$ of
goods for all $\gamma_i^j$ can be obtained as $G = (\gamma_i^j)$ with $\gamma_i^j > 0$ for $j = \{1, \ldots, m\}$ and $i = \{1, \ldots, n\}$.

$$G = \begin{pmatrix}
\gamma_1^1 & \gamma_1^2 & \cdots & \gamma_1^m \\
\gamma_2^1 & \gamma_2^2 & \cdots & \gamma_2^m \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_n^1 & \gamma_n^2 & \cdots & \gamma_n^m
\end{pmatrix}$$

Now $\gamma_i^j$ and $\nu_i^j$ can be used to determine, whether some individual $i$ is to be considered as undersupplied, supplied or oversupplied with regard to a specific dimension of need. From this classification the subsets $U$, $S$ and $O$ can be obtained from the set $P$. An individual is considered as undersupplied with regard to some good, if his endowment in this dimension is smaller than his corresponding need. It is considered as supplied with regard to some good, if his endowment equals his need in this dimension. Finally it is considered as oversupplied with regard to some good, if his endowment is greater than his corresponding need.

At this point a first classification of individuals can be undertaken: According to the union definition of the multidimensional measurement of poverty it can now be stated that every individual that suffers some undersupply in at least one dimension counts as undersupplied in general. Those whose needs are exactly met in every dimension count as supplied and those whose needs are oversupplied in at least one dimension while not suffering undersupply in any other count as oversupplied.

**Definition 1 (Undersupply)** An individual $i$ is undersupplied in some dimension of need $j$ if $\gamma_i^j < \nu_i^j$. The set of undersupplied individuals contains every individual that is undersupplied in at least one dimension; it is denoted as $U = \{i \in P : \exists j (\gamma_i^j < \nu_i^j)\}$; their number is given by $u = \#(U)$.

**Definition 2 (Supply)** An individual $i$ is supplied in some dimension of need $j$ if $\gamma_i^j = \nu_i^j$. The set of supplied is $S = \{i \in P : \forall j (\gamma_i^j = \nu_i^j)\}$; their number is given by $s = \#(S)$.

**Definition 3 (Oversupply)** An individual $i$ is oversupplied in some dimension of need $j$ if $\gamma_i^j > \nu_i^j$. The set of oversupplied is $O = \{i \in P : \forall j (\gamma_i^j \geq \nu_i^j)\}$; their number is given by $o = \#(O)$.  

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3 A First Multidimensional Measure of Need-Based Justice

In a first step a subtraction of the matrices can be performed with $G - N$, as follows, to obtain the status of supply for every individual in every dimension that is represented.

\[
\begin{pmatrix}
\gamma_1 & \gamma_1 & \ldots & \gamma_1^n \\
\gamma_2 & \gamma_2 & \ldots & \gamma_2^n \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_n & \gamma_n & \ldots & \gamma_n^n
\end{pmatrix}
- 
\begin{pmatrix}
\nu_1 & \nu_1 & \ldots & \nu_1^n \\
\nu_2 & \nu_2 & \ldots & \nu_2^n \\
\vdots & \vdots & \ddots & \vdots \\
\nu_n & \nu_n & \ldots & \nu_n^n
\end{pmatrix}
\]

This gives out a negative value for undersupply, a zero for supply and a positive value for oversupply, providing a first impression of the supply situation across all considered dimensions of need for every individual.

For illustration a set $P$ of individuals $i = \{1, 2, 3\}$ is assumed. Every individual exhibits individual needs $\nu_i^j$ that are expressed in four different dimensions $j = \{1, 2, 3, 4\}$, take for example: thirst, hunger, security and social acceptance. This results in a matrix $N$ of needs for all $\nu_i^j$ with $N = (\nu_i^j)$. For simplification, it is assumed that every individual has the same quantification of need – say five units – across all four dimensions.

\[
N = \begin{pmatrix}
5 & 5 & 5 & 5 \\
5 & 5 & 5 & 5 \\
5 & 5 & 5 & 5
\end{pmatrix}
\]

In addition it is assumed that there are different goods for the satisfaction of different needs that are allocated heterogeneous among those individuals and their needs. This may give us a matrix $G$ of goods for all $\gamma_i^j$ with $G = (\gamma_i^j)$ as follows.

\[
G = \begin{pmatrix}
3 & 2 & 0 & 0 \\
5 & 5 & 5 & 5 \\
3 & 2 & 7 & 8
\end{pmatrix}
\]

If those matrices are subtracted with $G - N$ this gives some insight in the status of supply for every individual in every dimension of need.
$G - N = \begin{pmatrix} -2 & -3 & -5 & -5 \\ 0 & 0 & 0 & 0 \\ -2 & -3 & 2 & 3 \end{pmatrix}$

Recalling the above stated definitions it becomes clear that the first individual is undersupplied in every dimension, while the second one is overall supplied. The third one however suffers undersupply in two dimensions while being oversupplied in two others. According to the definitions above the subsets of the undersupplied ($U$), supplied ($S$), and oversupplied ($O$) individuals in $P$ can be obtained.

$$i \in \begin{cases} 
U & \text{if } i \in P : \exists j (\gamma^j_i < \nu^j_i) \\
S & \text{if } i \in P : \forall j (\gamma^j_i = \nu^j_i) \\
O & \text{if } i \in P : \exists j (\gamma^j_i < \nu^j_i) \land \exists j (\gamma^j_i > \nu^j_i) 
\end{cases}$$

Therefore, none of the three individuals counts as oversupplied, although one is oversupplied in two dimensions of need. One individual counts as supplied and the others as undersupplied.

As for the one-dimensional measurement of need-based justice this can be aggregated to some multidimensional index $M$ of need-based justice. It would, for example, be possible to transfer some multidimensional enhancement for the poverty measure of Foster and colleagues as stated by Kockläuner (2012). Nonetheless another approach is to be carry out here.

First of all, a basic assumption has to be recalled: In every case it should make a difference whether a person gains or looses some of its endowment, as long as her need stays the same. This shall be captured by the measure; depicting a monotonicity of justice evaluation that is gradually dependent of the supply an individual has in some dimension of need (Bauer 2018). The justice evaluation of the measure – let it be denoted $M(\nu, \gamma)$ – can therefore be understood as a monotonic function. It can be distinguished between several forms of monotonicity: The measure can be strictly increasing or strictly decreasing. The former shall be the case if a greater endowment leads to a greater justice of the distribution; so if $\gamma^j_i < \gamma^j_i'$ then $M(\nu, \gamma) > M(\nu, \gamma')$. The latter shall be the case if a greater endowment leads to a lower justice of the distribution; so if $\gamma^j_i < \gamma^j_i'$ then $M(\nu, \gamma) < M(\nu, \gamma')$. Whereby the measure $M$ as a justice evaluation function can be sectionally defined with different monotonic properties.

Why is this sectional definition important? For the case of undersupply it seems pretty clear what kind of monotonicity shall be required: As long as
comparative considerations are left aside (Feinberg 1974, Springhorn 2017) and it therefore has not to be dealt with inequality among the considered individuals, it can be assumed that an additional unit of some good states an approximation to the legitimate need as long as the individual was initially undersupplied. Therefore, the measure should be strictly monotonically increasing for the case of undersupply. Now how about the case of oversupply? Shall the measure there too be strictly monotonically increasing or should it instead be strictly decreasing? An if so, why?

For example and for the sake of the argument it shall be considered that an exact alignment of need and allocation constitutes an ideal state of distribution. Therefore, the measure shall be strictly decreasing in the case of oversupply. This could lean on conceptions known from classical antiquity, for example, the μεσότης (mesotes) in the work of Aristotle, or recent debates about sufficiency. This in mind the following could be adopted for the measurement of needs-based-justice:

$$M(\nu, \gamma) = \frac{1}{m} \sum_{j=1}^{m} \prod_{i=1}^{n} \left( 1 - \frac{\left| (\gamma^j_i - \nu^j_i) \right|}{\gamma^j_i + \nu^j_i} \right)$$ (3)

In (3) a similarity measure, namely the earth similarity index as proposed by Schulze-Makuch and colleagues (2011), is transformed to measure the similarity of the actual allocation to the proclaimed needs, summing it up for every individual divided by their whole number. The index from Schulze-Makuch and colleagues states a variation of the similarity index from Bray and Curtis (Bray and Curtis 1957, Bloom 1981). It gives a number between 0 – for no similarity – and 1 – for being identical – for the comparison of two states.

4 Conclusion

It has been pointed out that one-dimensional measures of need-based justice struggle with a conceptual problem, that can be overcome by constructing multidimensional measures. One example of a possible index has been shortly introduced.

This attempt could be pushed even further, allowing for the construction of a general index for distributive justice, incorporating, for example, the four

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2 Alternative assumptions are covered in Bauer (2018).
dimensions of equality ($\varepsilon$), efficiency ($\mu$), desert ($\delta$) and need ($\nu$), combining them in a single matrix $D$ as follows:

$$D = \begin{pmatrix} \varepsilon_1 & \mu_1 & \delta_1 & \nu_1^1 & \cdots & \nu_1^m \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \varepsilon_n & \mu_n & \delta_n & \nu_n^1 & \cdots & \nu_n^m \end{pmatrix}$$

Those dimensions could be split up into different sub-dimensions, as depicted above for the category of need – which, of course, may lead to some interesting new areas of inquiry.

References


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